Method for calculating of service characteristics with typical logistic metrics

Todor Todorov[1] and Marin Georgiev[2]

Faculty of Mechanical Engineering, Technical University of Sofia (TU-Sofia),
8, Kliment Ohridski St., Sofia-1000, Bulgaria
[1]**t_todorov@tu-sofia.bg**
[2]**mgeor@tu-sofia.bg**

An important question in the preliminary dimensioning of processes of the technical logistics in warehouses is the calculating of the probability characteristics of the serving time with non-stationary hoisting and hauling devices (for example: transmanipulators in high-bay warehouses and electric trucks in conventional warehouses). The known works from the last 30 years analyze separately the problems in the specifics orthogonal (Manhattan) metric and metric of Chebyshev.

In the present material a general method is being introduced. This method offers a way for calculating of base probability valuations of the serving time in deprived of individuality space with uniform distribution of the requests by a movable server through single command cycle in simple and complex work zones in the typical for the intralogistics metrics. The method is based on isochrone analysis through geometrical interpretation of the probability distributions. Applicable examples are also included.

Keywords: warehouse logistics, metrics, service meantime, isochrones

Introduction

The problem for calculating of service time characteristics (through cyclic hoisting and hauling devices) is one of the longest and most exhaustive examined ones by the analytical logistics [1], [2], [3] and others. In one of its fundamental varieties the problem is often formulated for a server, which is non-stationary, but it should move to a point in the plane, where a request for service has been arisen. When the position has been reached an action with defined duration (determined or accidental) is being performed (for example – storaging or retrieving of warehouse unit). This action may complete the request at the same position, or (more often) a secondary movement of the server back to the start position is necessary, where the request is being completed.

The described process is named *single command cycle* [2], [4], and it’s typical as for traditional tasks in the warehouse logistics, also for a number of tasks met in the daily round of servicing, trade and the industry – emergency service, courier service, fire departments and others.

The movement of the server is being characterized by its constructive specialities, and also by the service zone geometry. The problem has been solved for the typical intralogistics metrics – orthogonal metric and Chebyshev’s metric [3], [4]. The researches of the last 20 years are being analyzed in a current work [5]. The existing methods solve the problem separately depending by the metric (orthogonal or Chebyshev’s one) in which the server works.

The present material has the purpose to present a general method, which draws attention to the given geometry and metric, but doesn’t make the problem in a special case because of them.

Method’s presentation

In the present material are being investigated the calculations related to the probability characteristics of the server movement times. The times necessary for loading, unloading, storaging and retrieving are being ignored, corresponding to the work of Bozer and White [5].

For the method’s presentation a rectangular service zone (rack) will be used. It’s being serviced by a movable server (transmanipulator). Let the requests are uniformly distributed in the zone. Because the traveling times are of interest, the service zone is at first transformed from the (original) space domaing to a time domain. Then the time domain will be transformed to a normalized time domain (this approach has been shown by Goetschalckx). The transformations are illustrated on *Figure 1*.

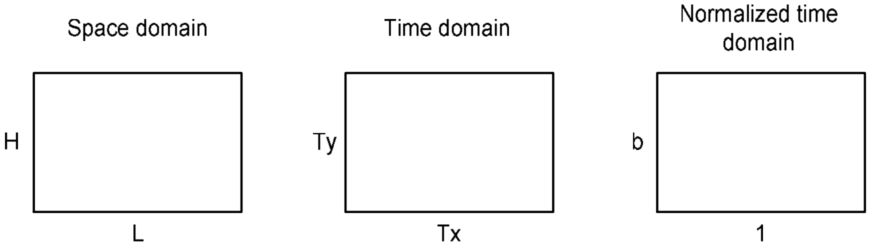

Figure 1

The space domain represents the service zone with its real size – length L and height H. Through division by the server velocity, the corresponding horizontal and vertical sizes are: $T_x = L / v_x$ and $T_y = H / v_y$.

Through the time domain sizes, the maximal traveling time in the zone T is being calculated. In the case of *Chebyshev's* metric, and choice of loading-unloading station at the lower-left corner of the zone, for T is valid: $T=max\{T_x,T_y\}$.

The sizes of the normalized time domain are being obtained as the corresponding sizes of the time domain are being divided by the maximal time T – i.e. horizontal $T_v/T=1$, *and vertical* $T_y/T=b$. For values of b: $0 < b < 1$.

As first step of the method, is being calculated the expected traveling time from the loading-unloading station to a random request point in the normalized time domain. For a point with coordinates (x,y) in the domain, the traveling time z is $z = max(x,y)$. The mean time is being calculated as integral sum of the product of the traveling time and the probability for request, for each point of the normalized domain S:

$$E(z) = \int_S z.p(z) \qquad (1)$$

For this calculation to be done, the domain points are being considered in groups. On the normalized time domain, the isochrones are being examined, with epicenter the loading-unloading station (point 0). The distance between point 0 and each isochrone is exactly z *(0≤z≤1)* – the traveling time (*Figure 2*). If the isochrone is considered as infinitesimal surface, then the probability of a random request to lie on a given isochrone (when the requests are uniformly distributed) equals to the ratio of the isochrone surface *dW(z)* to the whole area of the service zone.

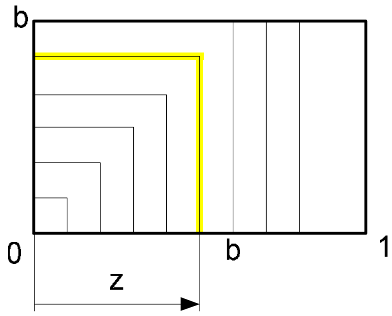

Figure 2

$$p(z) = \frac{dW(z)}{S}, \text{ за } 0 \le z \le 1 \qquad (2)$$

After substitution of equation (2) into (1), for the mean traveling time E(z) the following integral sum is obtained:

$$E(z) = \int_0^1 z \frac{dW(z)}{S} \qquad (3)$$

The equation represents a sum of the geometrical probabilities *dW(z)/S*, weighted according z – the distance from the isochrone surface *dW(z)* to the epicenter of the isochrones.

But for *E(z)* is also known:

$$E(z) = \int_{-\infty}^{+\infty} z.f(z)dz = \int_0^1 z.f(z)dz \qquad (4)$$

From (3) and (4) follows: $z.f(z)dz = z\frac{dW(z)}{S} \Rightarrow f(z)dz = \frac{dW(z)}{S}$

The last transformation has been done, because the factor z changes both sides equally. Through the last equation the probability mass function may be obtained:

$$f(z)=\frac{dW(z)}{S.dz}\text{, за } 0\le z\le 1 \qquad (5)$$

The application of this equation is shown below. At this moment it's enough to note, that through geometrical probability it's possible to calculate the probability mass function f(z). The next steps are calculation of the variance and the variation coefficient of z.

$$D(z)=E(z^2)-E(z)^2 \qquad (6)$$

$$E(z^2)=\int_0^1 z^2.f(z)dz \qquad (7)$$

$$c(z)=\frac{\sqrt{D(z)}}{E(z)} \qquad (8)$$

By knowing the probability characteristics of the random variable z, it's easy to obtain the characteristics of the single command cycle. The service time of cycle for a request residing on a given isochrone equals to the time necessary for traveling to, and back traveling from this isochrone. I.e. the time SC for single command cycle equals to the doubled time z, which is traveling time from the point 0 to the isochrone. Or from mathematical point of view, SC is a function of the random variable z:

$$SC=s(z)=2z \qquad (9)$$

Then from the known dependences it can be written:

$$E(SC)=\int_0^1 s(z).f(z)dz=\int_0^1 2z.f(z)dz \qquad (10)$$

$$D(SC)=E(s(z)^2)-E(s(z))^2 \qquad (11)$$

$$SC(z)^2=SC(z).SC(z)=2z.2z=4z^2$$

$$E(SC(z)^2)=\int_0^1 SC(z)^2.f(z)dz=\int_0^1 4z^2.f(z)dz \qquad (12)$$

$$c(SC)=\frac{\sqrt{D(SC)}}{E(SC)} \qquad (13)$$

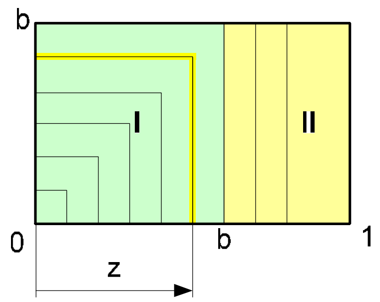

Figure 3

Thus, the probability characteristics of single command cycle have been calculated, based on equation (5). The metric has been used just as contrivance for defining the surfaces dW(z), and it doesn't influence in other way the method.

As it's being shown in the application of the method in the following points, the probability mass function *f(z)* is not ever continuous function. For this reason, the integral sum from 0 to 1 should be divided to intervals at the points of discontinuity. Such dividing is comparatively easy, because of the geometrical nature of the probabilities.

Application in Chebyshev's metric

Consider a rectangular zone with uniformly distributed service request. The zone is being serviced by a server, which works in a Chebyshev's metric, with equal horizontal and vertical velocities. Without lose of generality, assume that the zone's length L is greater than its height H.

By the given conditions the maximal traveling time in the zone is $T=L/v_x$. Through it, the normalized time domain is being obtained - a rectangle with sides $b=T_y/T$ and

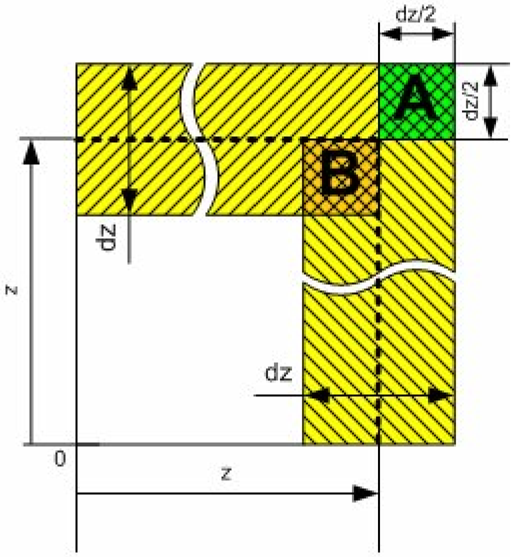

Figure 4

$1 = T_x / T$ (*Figure 3*).

On the normalized time domain are also the isochrones added, according to the rules of the Chebyshev's metric. Their epicenter is the loading-unloading station (point 0). The distance between point 0 and each isochrone is z ($0 \le z \le 1$), representing the traveling time.

In this case, the infinitesimal surfaces should be defined in two intervals – for z = [0,b], and z=(b,1]. In the first interval the isochrones are vertical and horizontal straight lines, each of them lying at time (distance) z from point 0. At the same time from point 0 lie all points at distance (in Chebyshev's metric) smaller than dz/2 from the straight lines, as $dz \to 0$. Grouped, these points represent the infinitesimal surface *dW(z)* of the isochrone, which should be calculated. The surface *dW(z)* is illustrated as hatched area on *Figure 4*. For representation of the surface as function of z, the following reasoning may be done: The double hatched areas A and B are equal quadrates, with sides dz/2. Then, the surface *dW(z)* may be considered as two rectangles – with sides z and dz – overlapped over the area B, and the area A to be ignored. Thus, for the surface *dW(z)* in the interval from 0 to b can be written *dW(z)=2.z.dz*.

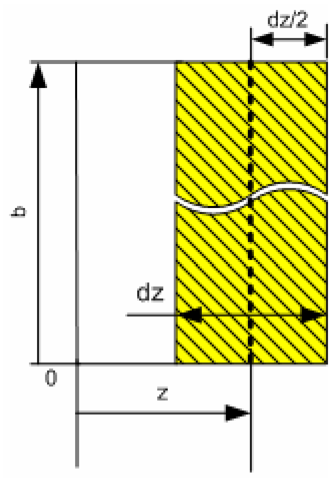

Figure 5

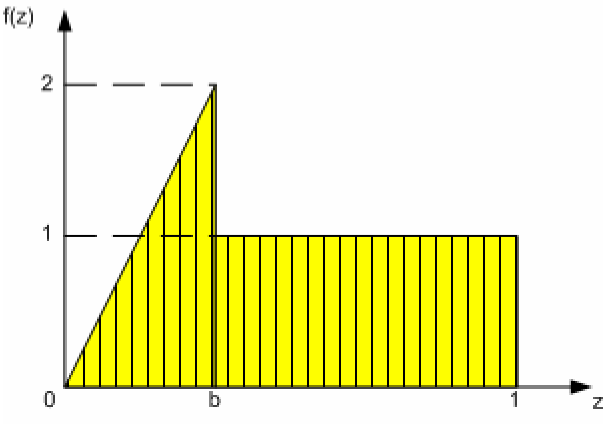

Figure 6

In the interval from b to 1, the isochrone represents a vertical straight line, which lies at time z from point 0. The set of points lying at time smaller than *dz/2* is an infinitesimal rectangle with sides z and dz (*Figure 5*). And the surface of the rectangle equals *dW(z)=b.dz*.

Summarizing for the whole zone z = [0,1], according to equation (5), the probability mass function can be written, as the surface of the whole zone is $S = 1.b = b$:

$$dW(z) = \begin{cases} 2zdz & 0 \le z < b \\ bdz & b < z \le 1 \end{cases} \Rightarrow f(z) = \frac{dW(z)}{S.dz} = \frac{dW(z)}{b.dz} = \begin{cases} \dfrac{2z.dz}{b.dz} = \dfrac{2z}{b} & 0 \le z < b \\ \dfrac{b.dz}{b.dz} = 1 & b < z \le 1 \end{cases}$$

The probability mass function f(z) coincides with the derived one in [4]. It has been illustrated graphically on *Figure 6*.. When the f(z) is already a known function, the probability characteristics of the variable z are being calculated through the equations (6)-(9) given in the previous point. For the probability characteristics of the single command cycle (which are of greater interest) the equations (10)-(13) are being used.

Common form of the characteristic
$E(SC) = \int_0^1 2z.f(z)dz = \int_0^b 2z.\frac{2z}{b}dz + \int_b^1 2z.1.dz = \frac{b^2}{3} + 1$
$E(SC^2) = \int_0^1 4z^2.f(z)dz = \int_0^b 4z^2.\frac{2z}{b}dz + \int_b^1 4z^2.1.dz = \frac{2b^3}{3} + \frac{4}{3}$
$D(SC) = E(SC^2) - E(SC)^2 = -\frac{b^4}{9} + \frac{2b^3}{3} - \frac{2b^2}{3} + \frac{1}{3}$
$c(SC) = \frac{\sqrt{D(SC)}}{E(SC)} = \frac{\sqrt{-b^4 + 6b^3 - 6b^2 + 3}}{b^2 + 3}$

Table 1

It should be noted, that the derived expressions coincide with cited results of other authors [5]. The common forms of these equations have been given in *Table 1*. An attention should be paid, that all results are in normalized time domain. The real time values can be calculated through T.

Application in orthogonal metric

In this point equation (5) is being used again, in order that the mean service time to be calculated in a service zone with orthogonal metric.

Consider a rectangular zone with uniformly distributed service request. The zone is being serviced by a server, which works in a Chebyshev's metric, with equal horizontal and vertical velocities. Without lose of generality, assume that the zone's length L is greater than its height H.

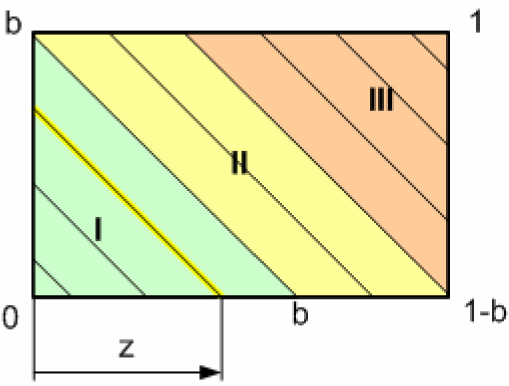

Figure 7

For the given conditions the maximal traveling time (the maximal distance in the time domain) is $T = H / v_H + L / v_L$, which is the necessary time for traveling from the lower-left corner, to the upper-right one. Through the time T the normalized time domain is being constructed – a rectangle with sides $b = T_y / T$ and $1-b = T_x / T$, for b≤0,5. In the model the isochrones (the infinitesimal surfaces) are being illustrated, which in orthogonal metric represent straight lines sloped at angle 45° towards the rectangle sides (*Figure 7*).

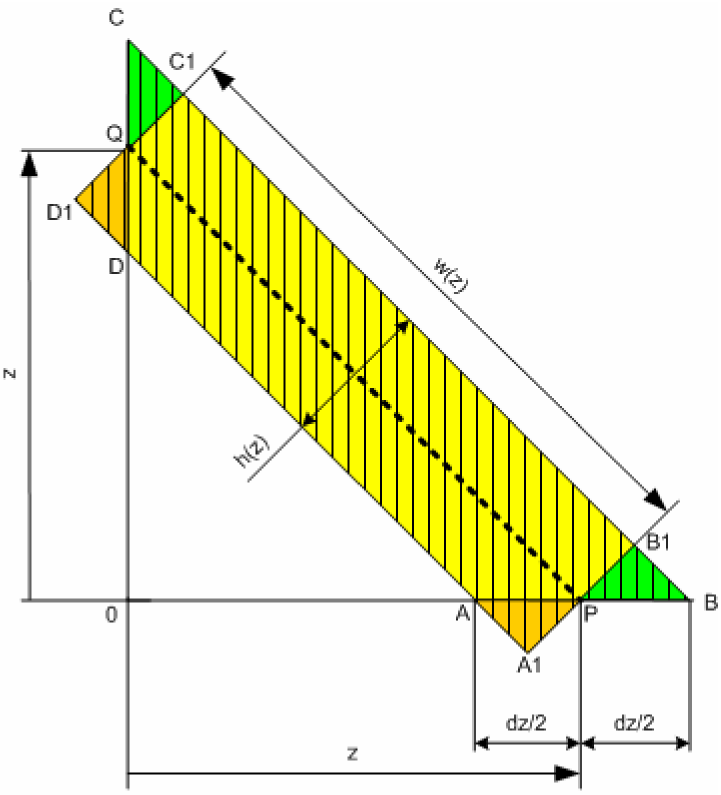

Figure 8

In this case the infinitesimal surfaces *dW(z)* should be considered in three intervals: Interval I: $z \in [0, b]$; Interval II: $z \in (b, 1-b]$ and Interval III: $z \in [1-b, 1]$.

An isochrone from interval I is being considered (*Figure 8*). It represents isosceles trapezoid ABCD, with middle segment PQ which lies on distance z from point 0. The distance between the middle segments of two neighbour isochrones is dz (in orthogonal metric). Thus, the length of the congruent segments is dz, and the points P and Q are their middles. Through the point P and Q the heights A1B1 and C1D1 are being constructed. The triangles PBB1 and PAA1 (and also QCC1 and QDD1) are congruent. If from the ABCD trapezoid's surface *dW(z)* subtract the surfaces of the triangles PBB1 and QCC1, and then add the surfaces of triangles PAA1 and QDD1, the result will be same. Then, the surface *dW(z)* is also equal to the surface of the rectangle A1B1C1D1. The rectangle itself has sides *w(z)* and *h(z)*, for which it's easy to obtain:

$$w(z) = z\sqrt{2} \quad h(z) = \frac{dz}{\sqrt{2}}$$

Thus, for the isochrone surfaces in the interval $z \in [0, b]$ it can be written:

$$dW^{I}(z) = w(z).h(z) = \frac{z\sqrt{2}dz}{\sqrt{2}} = zdz$$

Througn similar reasonings for intervals II и III the following is derived: $dW^{II}(z) = b.dz$ and $dW^{III}(z) = (1-z)dz$.

	$0 \le z < b$	$b < z \le 1-b$	$1-b < z \le 1$
dW(z)	zdz	bdz	$(1-z)dz$
f(z)	$\frac{z}{b.(1-b)}$	$\frac{1}{(1-b)}$	$\frac{1-z}{b.(1-b)}$

Table 2

Through these results according to equation (5), the probability mass function f(z) can be written, as taking into account that $S = b.(1-b)$. The result has been given in *Table 2*. Graphically, the function f(z) is shown on *Figure 9*.

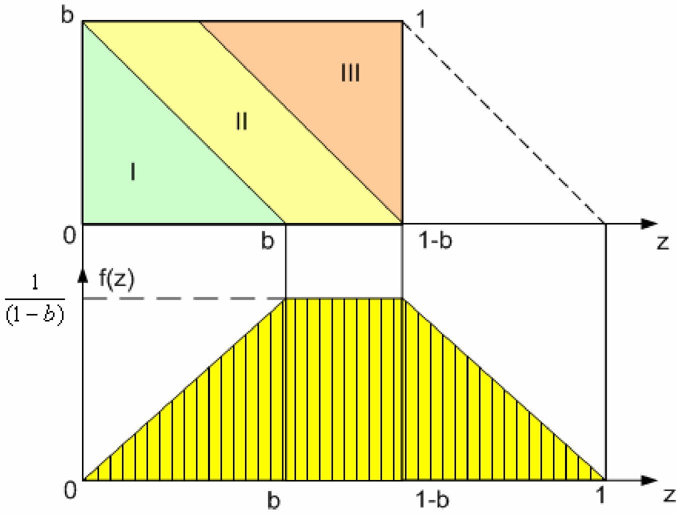

Figure 9

Again, when the probability mass function f(z) is known, the probability characteristics of the traveling time z can be obtained. And also, through representation of the service time as function of random variable SC(z), the characteristics of the single command cycle service time can be calculated. For example:

$$E(SC) = \int_0^1 2z.f(z)dz =$$

$$= \int_0^b \frac{2z.z}{b(1-b)}dz + \int_b^{1-b} \frac{2z.1}{(1-b)}.dz + \int_{1-b}^1 \frac{2z(1-z)}{b(1-b)}.dz = 1$$

The same result will be produced for the case with orthogonal metric, trough the approach given in [3]. The other characteristics are given in *Table 3*, as their calculation has been done with equations (10)-(13).

It's a good idea to note again, that the values are valid in normalized time domain. To obtain the real time values, each of them should be multiplied by T.

Common form of the characteristic
$E(SC^2) = \int_0^1 4z^2.f(z)dz = \frac{2}{3}b^2 - \frac{2}{3}b + \frac{4}{3}$
$D(SC) = E(SC^2) - E(SC)^2 = \frac{2}{3}b^2 - \frac{2}{3}b + \frac{1}{3}$
$c(SC) = \frac{\sqrt{D(SC)}}{E(SC)} = \frac{1}{3}\sqrt{6b^2 - 6b + 3}$

Table 3

Conclusion

The method presented gives a general approach for solving problems of calculating probability characteristics of the service time, through a direct geometrical analogy of the isochrones, independent of the metric type. The application of this method for single command cycle has been presented for different metrics, and also coincidence with the results from other authors is proved. The approach for using isochrones through direct geometrical analogy, as contrivance for obtaining the service time characteristics, is applicable for arbitrary convex service zones, and also for non-convex, with some conditions about the location of the base position.

Literature Survey

1. Gudehus,T. Grundlagen der Spielzeitberechnung für automatische Hochregallager. Deutsche Hebe- und Fördertechnik (Sonderheft) 1972, 18, S. 63-68
2. Gudehus,T. Grundlagen der Kommissioniertechnik : Dynamik der Lager- und Warenverteilsysteme, Verlag W.Girardet, 1973
3. Grosseschallau,W. Materialflußrechnung, Springer V., 1984
4. Bozer, Y., J.A.White. Travel-Time Models for Automated Storage/Retrieval Systems. IEE Transactions 1984, 16 (4), S.329-338.
5. Lippolt,C. Spielzeiten in Hochregallagern mit doppeltiefer Lagerung, Dissertation, Wissenschaftliche Berichte des Insitutes für Fördertechnik und Logistiksysteme der Universität Karlsruhe, Mai 2003, Band 58